Questions

Q1.

(a) Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of $\left(2-\frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7

(1)

(Total for question = 5 marks)

Q2.

(a) Find the first 3 terms, in ascending powers of *x*, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$\mathrm{f}(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9$, where *a* and *b* are constants

Given that the first two terms, in ascending powers of *x*, in the series expansion of f(*x*) are 128 and 36*x*,

(b) find the value of *a*,

(2)

(c) find the value of *b*.

(2)

(Total for question = 8 marks)

Q3.

(a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6

You do not need to perform the calculation.

(1)

(Total for question = 5 marks)

Q4.

(a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient x^3 is 3 times the coefficient of x,

(b) find the possible values of k.

(3)

(Total for question = 6 marks)

Q5.

$$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of g(x) is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

(Total for question = 7 marks)

Q6.

In the binomial expansion of

$(a + 2x)^7$ where a is a constant

the coefficient of x^4 is 15 120

Find the value of a.

(Total for question = 3 marks)

Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	$\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6.\left(-\frac{x}{2}\right)+\binom{7}{2}2^5.\left(-\frac{x}{2}\right)^2+...$	M1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = 128+...$	B1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... -224x +...$	A1	1.1b
	$\left(2-\frac{x}{2}\right)^7 = ... + ... + 168x^2 \ (+...)$	A1	1.1b
		(4)	
(b)	Solve $\left(2-\frac{x}{2}\right) = 1.995$ so $x = 0.01$ and state that 0.01 would be substituted for x into the expansion	B1	2.4
		(1)	
			(5 marks)

Notes

(a) M1: Need correct binomial coefficient with correct power of 2 and correct power of x. Coefficients may be given in any correct form; e.g. 1, 7, 21 or ${}^7C_0$, ${}^7C_1$, ${}^7C_2$ or equivalent

B1: Correct answer, simplified as given in the scheme.

A1: Correct answer, simplified as given in the scheme.

A1: Correct answer, simplified as given in the scheme.

(b) B1: Needs a full explanation i.e. to state $x = 0.01$ **and** that this would be substituted **and** that it is a solution of $\left(2-\frac{x}{2}\right) = 1.995$

Q2.

Question	Scheme	Marks	AOs
(a)	$\left(2-\frac{x}{16}\right)^9 = 2^9 + \binom{9}{1}2^8.\left(-\frac{x}{16}\right)+\binom{9}{2}2^7.\left(-\frac{x}{16}\right)^2+...$	M1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = 512+...$	B1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ... -144x +...$	A1	1.1b
	$\left(2-\frac{x}{16}\right)^9 = ... + ... + 18x^2 (+ ...)$	A1	1.1b
		(4)	
(b)	Sets $'512'a = 128 \Rightarrow a = ...$	M1	1.1b
	$(a=)\frac{1}{4}$ oe	A1 ft	1.1b
		(2)	
(c)	Sets $'512'b + '-144'a = 36 \Rightarrow b = ...$	M1	2.2a
	$(b=)\frac{9}{64}$ oe	A1	1.1b
		(2)	
		(8 marks)	

(a) alt	$\left(2-\frac{x}{16}\right)^9 = 2^9\left(1-\frac{x}{32}\right)^9 = 2^9\left(1+\binom{9}{1}\left(-\frac{x}{32}\right)+\binom{9}{2}\left(-\frac{x}{32}\right)^2+...\right)$	M1	1.1b
	$= 512+...$	B1	1.1b
	$= ... -144x +...$	A1	1.1b
	$= ... + ... + 18x^2 (+ ...)$	A1	1.1b

Notes

(a)

M1: Attempts the binomial expansion. May be awarded on either term two and/or term three Scored for a correct binomial coefficient combined with a correct power of 2 and a correct power of $\left(\pm\frac{x}{16}\right)$ Condone $\binom{9}{2}2^7.\left(-\frac{x^2}{16}\right)$ for term three.

Allow any form of the binomial coefficient. Eg $\binom{9}{2} = {}^9C_2 = \frac{9!}{7!2!} = 36$

In the alternative it is for attempting to take out a factor of 2 (may allow 2^n outside bracket) and having a correct binomial coefficient combined with a correct power of $\left(\pm\frac{x}{32}\right)$

B1: For 512

A1: For $-144x$

A1: For $+\ 18x^2$ Allow even following $\left(+\frac{x}{16}\right)^2$

Listing is acceptable for all 4 marks

(b)

M1: For setting their $512a=128$ and proceeding to find a value for a. Alternatively they could substitute $x=0$ into both sides of the identity and proceed to find a value for a.

A1 ft: $a=\frac{1}{4}$ oe Follow through on $\frac{128}{\text{their } 512}$

(c)

M1: Condone $512b \pm 144 \times a = 36$ following through on their 512, their -144 and using their value of "a" to find a value for "b"

A1: $b=\frac{9}{64}$ oe

Q3.

Question	Scheme	Marks	AOs
(a)	2^6 or 64 as the constant term	B1	1.1b
	$\left(2+\frac{3x}{4}\right)^6 = ...+{}^6C_1 2^5\left(\frac{3x}{4}\right)^1+{}^6C_2 2^4\left(\frac{3x}{4}\right)^2+...$ $=...+6\times 2^5\left(\frac{3x}{4}\right)^1+\frac{6\times 5}{2}\times 2^4\left(\frac{3x}{4}\right)^2+...$	M1 A1	1.1b 1.1b
	$=64+144x+135x^2+...$	A1	1.1b
		(4)	
(b)	$\frac{3x}{4}=-0.075 \Rightarrow x=-0.1$ So find the value of $64+144x+135x^2$ with $x=-0.1$	B1ft	2.4
		(1)	
			(5 marks)

Notes
(a) **B1:** Sight of either 2^6 or 64 as the constant term **M1:** An attempt at the binomial expansion. This may be awarded for a correct attempt at either the second **OR** third term. Score for the correct binomial coefficient with the correct power of 2 and the correct power of $\frac{3x}{4}$ condoning slips. Correct bracketing is not essential for this M mark. Condone ${}^6C_2 2^4 \frac{3x}{4}^2$ for this mark **A1:** Correct (unsimplified) second **AND** third terms. The binomial coefficients must be processed to numbers /numerical expression e.g $\frac{6!}{4!2!}$ or $\frac{6\times 5}{2}$ They cannot be left in the form ${}^6C_1$ and/or $\binom{6}{2}$ **A1:** $64+144x+135x^2+...$ Ignore any terms after this. Allow to be written $64, 144x, 135x^2$ **(b)** **B1ft:** $x=-0.1$ or $-\frac{1}{10}$ **with** a comment about substituting this into their $64+144x+135x^2$ If they have written (a) as $64, 144x, 135x^2$ candidate would need to say substitute $x=-0.1$ into the sum of the first three terms. As they do not have to perform the calculation allow Set $2+\frac{3x}{4}=1.925$, solve for x and then substitute this value into the expression from (a) If a value of x is found then it must be correct

Alternative to part (a)

$$\left(2+\frac{3x}{4}\right)^6 = 2^6\left(1+\frac{3x}{8}\right)^6 = 2^6\left(1+{}^6C_1\left(\frac{3x}{8}\right)^1 + {}^6C_2\left(\frac{3x}{8}\right)^2 + ...\right)$$

B1: Sight of either 2^6 or 64

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term. Score for the correct binomial coefficient with the correct power of $\frac{3x}{8}$ Correct bracketing is not essential for this mark.

A1: A correct attempt at the binomial expansion on the second and third terms.

A1: $64+144x+135x^2+...$ Ignore any terms after this.

Q4.

Question	Scheme	Marks	AOs
(a)	$(1+kx)^{10} = 1+\binom{10}{1}(kx)^1+\binom{10}{2}(kx)^2+\binom{10}{3}(kx)^3...$	M1 A1	1.1b 1.1b
	$=1+10kx+45k^2x^2+120k^3x^3...$	A1	1.1b
		(3)	
(b)	Sets $120k^3 = 3\times 10k$	B1	1.2
	$4k^2 = 1 \Rightarrow k = ...$	M1	1.1b
	$k = \pm\frac{1}{2}$	A1	1.1b
		(3)	
			(6 marks)

(a)

M1: An attempt at the binomial expansion. This may be awarded for either the second or third term or fourth term. The coefficients may be of the form ${}^{10}C_1$, $\binom{10}{2}$ etc or eg $\frac{10\times 9\times 8}{3!}$

A1: A correct unsimplified binomial expansion. The coefficients must be numerical so cannot be of the form ${}^{10}C_1$, $\binom{10}{2}$. Coefficients of the form $\frac{10\times 9\times 8}{3!}$ are acceptable for this mark.

The bracketing must be correct on $(kx)^2$ but allow recovery

A1: $1+10kx+45k^2x^2+120k^3x^3...$ or $1+10(kx)+45(kx)^2+120(kx)^3...$
Allow if written as a list.

(b)

B1: Sets their $120k^3 = 3\times$ their $10k$ (Seen or implied)
For candidates who haven't cubed allow $120k = 3\times$ their $10k$
If they write $120k^3x^3 = 3\times$ their $10kx$ only allow recovery of this mark if x disappears afterwards.

M1: Solves a cubic of the form $Ak^3 = Bk$ by factorising out/cancelling the k and proceeding correctly to at least one value for k. Usually $k=\sqrt{\frac{B}{A}}$

A1: $k=\pm\frac{1}{2}$ o.e ignoring any reference to 0

Q5.

Question	Scheme	Marks	AOs
(a)	$(2+ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5x^5$	M1 A1	1.1a 1.1b
	Sets $448a^5 = 3402 \Rightarrow a^5 = \frac{243}{32}$	M1	1.1b
	$\Rightarrow a = \frac{3}{2}$	A1	1.1b
		(4)	
(b)	Attempts either term. So allow for 2^8 or ${}^8C_4 2^4 a^4$	M1	1.1b
	Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$	dM1	2.1
	$= 256 + 5670 = 5926$	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: An attempt at selecting the correct term of the binomial expansion. If all terms are given then the correct term must be used. Allow with a missing bracket ${}^8C_5 2^3 ax^5$ and left without the binomial coefficient expanded

A1: $448a^5x^5$ Allow unsimplified but ${}^8C_5$ must be "numerical"

M1: Sets their $448a^5 = 3402$ and proceeds to $\Rightarrow a^k = \ldots$ where $k \in \mathbb{N}$ $k \neq 1$

A1: Correct work leading to $a = \frac{3}{2}$

(b)

M1: Finds either term required. So allow for 2^8 or ${}^8C_4 2^4 a^4$ (even allowing with *a*)

dM1: Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$

A1: cso 5926

Q6.

Question	Scheme	Marks	AOs
	${}^7C_4 a^3 (2x)^4$	M1	1.1b
	$\frac{7!}{4!3!} a^3 \times 2^4 = 15120 \Rightarrow a = ...$	dM1	2.1
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes:

M1: For an attempt at the correct coefficient of x^4.
The coefficient must have

- the correct binomial coefficient
- the correct power of a
- 2 or 2^4 (may be implied)

May be seen within a full or partial expansion.

Accept ${}^7C_4 a^3 (2x)^4$, $\frac{7!}{4!3!} a^3 (2x)^4$, $\binom{7}{4} a^3 (2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{4} a^3 16x^4$ etc.

or ${}^7C_4 a^3 2^4$, $\frac{7!}{4!3!} a^3 2^4$, $\binom{7}{4} a^3 2^4$, $35a^3 2^4$, $560a^3$ etc.

or ${}^7C_3 a^3 (2x)^4$, $\frac{7!}{4!3!} a^3 (2x)^4$, $\binom{7}{3} a^3 (2x)^4$, $35a^3(2x)^4$, $560a^3x^4$, $\binom{7}{3} a^3 16x^4$ etc.

or ${}^7C_3 a^3 2^4$, $\frac{7!}{4!3!} a^3 2^4$, $\binom{7}{3} a^3 2^4$, $35a^3 2^4$, $560a^3$

You can condone missing brackets around the "2x" so allow e.g. $\frac{7!}{4!3!} a^3 2x^4$

An alternative is to attempt to expand $a^7\left(1+\frac{2x}{a}\right)^7$ to give $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$

Allow M1 for e.g. $a^7\left(...\frac{7\times6\times5\times4}{4!}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...\binom{7}{4}\left(\frac{2x}{a}\right)^4...\right)$, $a^7\left(...35\left(\frac{2x}{a}\right)^4...\right)$ etc.

but condone missing brackets around the $\frac{2x}{a}$

Note that ${}^7C_3$, $\binom{7}{3}$ etc. are equivalent to ${}^7C_4$, $\binom{7}{4}$ etc. and are equally acceptable.

If the candidate attempts $(a + 2x)(a + 2x)(a + 2x)$… etc. then it must be a complete method to reach the required term. Send to review if necessary.

dM1: For $"560"a^3 = 15120 \Rightarrow a = ...$ Condone slips on copying the 15120 but their "560" must be an attempt at ${}^7C_4\times2$ or ${}^7C_4\times2^4$ and must be attempting the <u>cube root</u> of $\frac{15120}{"560"}$. **Depends on the first mark.**

A1: $a = 3$ and no other values i.e. ±3 scores A0

Note that this is fairly common:

$${}^7C_4 a^3 2x^4 = 70a^3x^4 \Rightarrow 70a^3 = 15120 \Rightarrow a^3 = 216 \Rightarrow a = 6$$

and scores M1 dM1 A0